the test solutions, and thus the degree of acidity or alkalescence was determined.

Diurnal variations in the acidity of the urine were observed. The acidity of the urine was found to ebb and flow; it was greatest a short time before food was taken, and was least about three hours after breakfast, and five or six hours after dinner, when it reached the minimum point; after which it again increased, and attained the maximum point previous to food being again taken.

If no food was taken, the acidity varied but slightly for twelve hours.

By comparing the effect of vegetable food with animal food, it appeared that the food which irritated the stomach most and caused most secretion of acid in the stomach, caused the greatest oscillations in the urine.

Dilute sulphuric acid taken in large doses produced but little effect on the variations of the acidity of the urine; but it was proved to increase the acidity of the urine.

Part II. *On the simultaneous variations of the amount of Uric Acid and the Acidity of the Urine in a healthy state.*

The result of these experiments is, that there is no relation between the acidity of the urine and the amount of uric acid in it. The urine that was most acid contained least uric acid; that which contained most uric acid was not most acid. All food causes an increase in the amount of uric acid in the urine; and there is no decided difference between vegetable and animal food, either as to the increase or diminution of the amount of uric acid in the urine.

Part III. *Variations of the Sulphates in the Urine in the healthy state, and on the influence of Sulphuric Acid, Sulphur and the Sulphates, on the Sulphates in the Urine.*

The result of these experiments is, that the sulphates in the urine are much increased by food, whether it be vegetable or animal. Exercise does not produce a marked increase in the sulphates. Sulphuric acid, when taken in large quantity, increases the sulphates in the urine. In small quantity, even when long-continued, no effect on the amount of sulphates is manifest.

Sulphur taken as a medicine increases the sulphates in the urine. Sulphate of soda and sulphate of magnesia produce the most marked increase in the sulphates in the urine.

February 8, 1849.

The EARL OF ROSSE, President, in the Chair.

A paper was read, entitled "On the application of the Theory of Elliptic Functions to the Rotation of a Rigid Body round a Fixed Point." By James Booth, L.L.D., F.R.S.

In the introduction to his investigation, the author, after noticing the investigations of D'Alembert and Euler, and the solution of this

problem by Lagrange, refers more particularly to the memoir of Poinsot, in which the motion of a body round a fixed point, and free from the action of accelerating forces, is reduced to the motion of a certain ellipsoid whose centre is fixed, and which rolls without sliding on a plane fixed in space; and likewise to the researches of Maccullagh, in which, by adopting an ellipsoid the reciprocal of that chosen by Poinsot, he deduced those results which long before had been arrived at by the more operose methods of Euler and Lagrange; observing, however, that it is to Legendre that we are indebted for the happy conception of substituting, as a means of investigation, an ideal ellipsoid having certain relations with the actually revolving body. He then states, that several years ago he was led to somewhat similar views, from remarking the identity which exists between the formulæ for finding the position of the principal axes of a body and those for determining the symmetrical diameters of an ellipsoid; and further observing that the expression for the perpendicular from the centre on a tangent plane to an ellipsoid, in terms of the cosines of the angles which it makes with the axes, is precisely the same in form as that which gives the value of the moment of inertia round a line passing through the origin. Guided by this analogy, he was led to assume an ellipsoid the squares of whose axes should be directly proportional to the moments of inertia round the coinciding principal axes of the body. This is also the ellipsoid chosen by Maccullagh. Although it may at first sight appear of little importance which of the ellipsoids—the *inverse* of Poinsot, or the *direct* of Maccullagh and the author—is chosen as the geometrical substitute for the revolving body, it is by no means a matter of indifference when we come to treat of the properties of the integrals which determine the motion. Generally those integrals depend on the properties of those curves of double flexure in which cones of the second degree are generally intersected by concentric spheres; and it so happens that the direct ellipsoid of moments is intersected by a concentric sphere in one of these curves. By means of the properties of these curves a complete solution may be obtained even in the most general cases, to which only an approximation has hitherto been made.

In the first section of the paper, the author establishes such properties as he has subsequently occasion to refer to, of cones of the second degree, and of the curves of double curvature in which these surfaces may be intersected by concentric spheres, some of which he believes will not be found in any published treatise on the subject. He considers that he has been so fortunate as to be the first to obtain the true representative curve of elliptic functions of the first order. It is shown that any spherical conic section, the tangents of whose principal semiarcs are the ordinates of an equilateral hyperbola whose transverse semiaxis is 1, may be rectified by an elliptic function of the first order; and the quadrature of such a curve may be effected by a function of the same order, when the cotangents of the halves of the principal arcs are the ordinates of the same equilateral hyperbola.

This particular species of spherical ellipse the author has called the "Parabolic Ellipse," because, as is shown in the course of the investigation, it is the gnomonic projection, on the surface of a sphere, of the common parabola whose plane touches the sphere at the focus. As in this species of spherical ellipse either the focus or the centre may be taken as the origin of the spherical radii vectores, in effecting the process of rectification, we are unexpectedly presented with Lagrange's scale of modular transformations, as also with the other equally well-known theorem by which the successive amplitudes are connected. Among other peculiar properties of the spherical parabolic ellipse established in this paper, it is shown that the portion of a great circle touching the curve, and intercepted between the perpendicular arcs on it from the foci, is always equal to a quadrant.

In the second and following sections, the author proceeds to discuss the problem which is the immediate subject of the paper. Having established the ordinary equations of motion, he shows that, if the direct ellipsoid of moments be constructed, the motion of a rigid body acted on solely by primitive impulses may be represented by this ellipsoid moving round its centre, in such a manner that its surface shall always pass through a point fixed in space. This point, so fixed, is the extremity of the axis of the plane of the impressed couple, or of the plane known as the invariable plane of the motion.

But a still clearer idea of the motion of such a body is presented in the subsequent investigations, it being there shown, that the most general motion of a body round a fixed point may be represented by a cone rolling with a certain variable velocity on a plane whose axis is fixed, while this plane revolves about its own axis with a certain uniform velocity. This cone may always be determined. For the circular sections of the invariable cone coincide with the circular sections of the ellipsoid of moments; whence the cyclic axes of the ellipsoid, or the diameters perpendicular to the planes of these sections, will be the focal lines of the supplemental cone; and as the invariable plane is always a tangent plane to this cone, we have sufficient elements given to determine it.

From these considerations it appears that we may dispense altogether with the ellipsoid of moments, and say that if two right lines be drawn through the fixed point of the body in the plane of the greatest and least moments of inertia, making angles with the axis of greatest moment, the cosines of which shall be equal to the square root of the expression

$$\frac{L(M-N)}{M(L-N)},$$

(L, M, N being the symmetrical moments of inertia round the principal axes) and a cone be conceived having those lines as focals, and touching moreover the invariable plane, the motion of the body will consist in the rotation of this cone on the invariable plane with a variable velocity, while the plane revolves round its own axis with an uniform velocity.

Although it is very satisfactory, the author remarks, in this way

to be enabled to place before our eyes, so to speak, the actual motion of the revolving body, yet it is not on such grounds that the paper is presented to this Society. It is as a method of investigation that it must rest its claims to the notice of mathematicians; as a means of giving simple and elegant interpretations of those definite integrals on the evaluation of which the dynamic state of a body at any epoch can alone be ascertained.

In these applications of the theory of elliptic functions, the author has been led to the remarkable theorem, that the length of the spiral, between two of its successive apsides, described in absolute space on the surface of a fixed concentric sphere, by the instantaneous axis of rotation, is equal to a quadrant of the spherical ellipse described on an equal sphere moving with the body, by the same instantaneous axis of rotation.

The last section of the paper is devoted to the discussion of that particular case in which the axis of the invariable plane is equal to the mean semiaxis of the ellipsoid of moments.

February 15, 1849.

W. R. GROVE, Esq., Vice-President, in the Chair.

A paper was in part read, entitled "Description of an Infusory Animalcule allied to the genus Notommata of Ehrenberg, hitherto undescribed." By John Dalrymple, Esq., F.R.C.S. Communicated by Thomas Bell, Esq., Sec. R.S.

February 22, 1849.

GEORGE RENNIE, Esq., Treasurer, Vice-President, in the Chair.

The Right Honourable Sir Francis Baring, Bart., First Lord of the Admiralty, was balloted for and duly elected into the Society.

The reading of a paper, entitled "Description of an Infusory Animalcule allied to the genus Notommata of Ehrenberg, hitherto undescribed." By John Dalrymple, Esq., F.R.C.S. Communicated by Thomas Bell, Esq., Sec. R.S., was resumed and concluded.

The examination of various specimens of the animalcule described by the author, disclosed the diœcious character of one of the more highly organized of the rotiferous class of Infusoria, hitherto supposed to be androgenous. This discovery was first made by observing the difference in the form and development of the embryo while still enclosed in the ovisac of the parent animal. From the extreme transparency of this form of rotifer, it is possible to trace the progressive development of the young from the Græffian vesicle in the ovary to the period of mature gestation, when the embryo is